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THE DEGREE OF COHERENCE OF RADIATING SYSTEMS AND THE DIRECTIONALITY OF LIGHT BEAMS

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FINAL REPORT

GRANT NO. DAAG29-76-G-0268

PREPARED BY

EMIL WOLF

DECEMBER 1976

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THE UNIVERSITY OF ROCHESTER
DEPARTMENT OF PHYSICS AND ASTRONOMY
ROCHESTER, NEW YORK

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This is a final report relating to research carried out under Grant DAAG29-76-G-0268. The duration of this grant was the four month period 6/30/76 - 10/29/76. The research was concerned with the characterization of optical wavefields generated by highly coherent sources and with the relationship between the specific intensity of radiation and a second-order correlation function of the field.

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ABSTRACT

This is a final report relating to research carried out under Grant DAAG29-76-G-0268. The duration of this grant was the four month period 6/30/76 - 10/29/76. The research was concerned with the characterization of optical wavefields generated by highly coherent sources and with a relationship between the specific intensity of radiation and a second-order correlation function of the field.

1. INTRODUCTION

The duration of this grant extended over the relatively short period of four months, from 6/30/76 to 10/29/76. Two lines of investigations, which are discussed in Section 2 of this report, were developed. The first of them concerns the statistical characterization of highly coherent light. The second concerns a theory proposed by V. I. Tatarskii as to the relationship between the specific intensity of radiation and a second-order correlation function of the field. This latter analysis was carried out in collaboration with Dr. Edward Collett of the U. S. Army Electronics Command, Fort Monmouth, New Jersey, and has lead to a complete clarification of the range of validity of Tatarskii's theory.

2. SUMMARY OF RESEARCH

(a) The Degree of Coherence of Coherent Fields

One of the basic concepts employed for characterizing the statistical properties of wavefields is the so-called complex degree of coherence, defined by the formula[1]

$$\gamma(\underline{r}_1, \underline{r}_2, \tau) = \frac{\Gamma(\underline{r}_1, \underline{r}_2, \tau)}{\sqrt{\Gamma(\underline{r}_1, \underline{r}_1, 0)}\sqrt{\Gamma(\underline{r}_2, \underline{r}_2, 0)}} . \tag{1}$$

In Eq. (1), $\Gamma(\underline{r}_1, \underline{r}_2, \tau)$ is the so-called mutual coherence function, defined as

$$\Gamma(\underline{r}_1, \underline{r}_2, \tau) = \langle V(\underline{r}_1, t+\tau) V^*(\underline{r}_2, t) \rangle , \tag{2}$$

where $V(\underline{r}_1, t)$ is the complex analytic signal that represents the light vibrations at the point $\underline{r}$ at time t and the sharp brackets denote a statistical average.

The normalization on the right-hand-side of Eq. (1) ensures that for all values of the arguments $\underline{r}_1$, $\underline{r}_2$ and τ,

$$|\gamma(\underline{r}_1, \underline{r}_2, \tau)| \leq 1 . \tag{3}$$

The two extreme cases, $\gamma = 0$ and $|\gamma| = 1$, represent complete (second-order) incoherence and complete (second-order) coherence respectively.

It is of importance, particularly in connection with analysis of radiation generated by lasers, to know the general form of the degree of coherence and of the mutual coherence function of a wavefield that is highly coherent. Several discussions of this problem appear in the published literature, but some of them are known to contain serious errors[2]. A rigorous treatment was given by Mehta, Wolf and Balachandran[3] who showed that if the field is completely coherent, in the sense that

$$|\gamma(\underline{r}_1, \underline{r}_2, \tau)| = 1 \tag{4}$$

for all possible values of the arguments of γ, then the complex degree of coherence is necessarily of the form

$$\gamma(\underline{r}_1, \underline{r}_2, \tau) = e^{i(\alpha - 2\pi\nu_0\tau)}, \tag{5}$$

where α and ν_0 are real constants and $\nu_0 > 0$. Together with (1) this result implies that for a fully coherent field, the mutual coherence function must necessarily have the factorized form

$$\Gamma(\underline{r}_1, \underline{r}_2, \tau) = U(\underline{r}_1) U^*(\underline{r}_2) e^{-2\pi i\nu_0\tau}. \tag{6}$$

While the results (5) and (6) are mathematically rigorous, they imply that a fully coherent field is a very degenerate field, for Eq. (6) leads at once to the conclusion that the spectrum of such a field must necessarily have a δ-function spectrum, (consisting of a single completely sharp line). It is clear that no *physically realizable* field, not even that generated by a laser, can have this property, since even laser light has a finite (even if only a relatively narrow) bandwidth. For this reason we have re-examined this problem.

The degree of coherence defined by Eq. (1) above is a measure of correlation in the space-time domain. It was pointed out in a recent paper[4] that for many purposes it might be preferable to employ a measure of correlation in the space-frequency domain and such a measure was, in fact introduced in that reference, in the following way: Let

$$W(\underline{r}_1, \underline{r}_2, \nu) = \int_{-\infty}^{\infty} \Gamma(\underline{r}_1, \underline{r}_2, \tau) e^{2\pi i\nu\tau} d\tau \tag{7}$$

be the cross-spectral density function of the field. It is known (cf. ref. 4) that the cross-spectral density function has also the significance of a correlation function; it characterizes the correlation

that exists between the Fourier components $v(\underline{r}_1,\nu)$ and $v(\underline{r}_2,\nu)$ of the field variable at the points $\underline{r}_1$ and $\underline{r}_2$ respectively. It was also shown in ref. 4 that the quantity

$$\mu(\underline{r}_1,\underline{r}_2,\nu) = \frac{W(\underline{r}_1,\underline{r}_2,\nu)}{\sqrt{W(\underline{r}_1,\underline{r}_1,\nu)}\,\sqrt{W(\underline{r}_2,\underline{r}_2,\nu)}} \tag{8}$$

may be considered to represent the complex degree of coherence in the space-frequency domain. This correlation coefficient was called the spectral degree of coherence and it was shown that

$$|\mu(\underline{r}_1,\underline{r}_2,\nu)| \leq 1 \tag{9}$$

for all values of the arguments of $\underline{r}_1$ and $\underline{r}_2$. The extreme cases $\mu = 0$ and $|\mu| = 1$ may be shown to represent complete incoherence and complete coherence respectively, of the field at the frequency component ν .

It follows at once from a positive definiteness property of the cross-spectral density function $W(\underline{r}_1,\underline{r}_2,\nu)$, established in reference 4, that the spectral degree of coherence is also positive definite, i.e., that for any positive frequency ν, any set of N position vectors $\underline{r}_1$, $\underline{r}_2$, ... $\underline{r}_N$ and any set of N complex numbers a_1, a_2, . . . a_N

$$\sum_{i=1}^{N}\sum_{j=1}^{N} a_i^* a_j\, \mu(\underline{r}_i,\underline{r}_j,\nu) \geq 0. \tag{10}$$

In our present investigation we have shown as a consequence of condition (10) that if the field is completely coherent at some particular frequency ν_0 in the sense that $|\mu(\underline{r}_1,\underline{r}_2,\nu_0)| = 1$ for all possible values of $\underline{r}_1$ and $\underline{r}_2$ then the spectral degree of coherence must necessarily be of the form

$$\mu(\underline{r}_1,\underline{r}_2,\nu_0) = e^{i[\phi(\underline{r}_1)-\phi(\underline{r}_2)]}, \tag{11}$$

where $\phi(\underline{r})$ is a real function of position. Moreover, it follows from Eqs. (11) and (8) that the cross-spectral density function of a field that is fully coherent at a frequency ν_0 has the form

$$W(\underline{r}_1, \underline{r}_2, \nu_0) = U(\underline{r}_1)\, U^*(\underline{r}_2), \tag{12}$$

where $U(\underline{r})$ is a (generally complex) function of position. Moreover, because of the fact that the mutual coherence function $\Gamma(\underline{r}_1, \underline{r}_2, \tau)$ satisfies two wave equations, the function $U(\underline{r})$ in (12) cannot be arbitrary, but in free space must satisfy the Helmholtz equation

$$\nabla^2 U(\underline{r}) + (2\pi\nu_0/c)^2\, U(\underline{r}) = 0. \tag{13}$$

Thus we have established the result that for a field to be completely coherent at a frequency ν_0, the cross-spectral density at that frequency must factorize in the form (12), and the function $U(\underline{r})$ must satisfy the Helmholtz equation (13), where c denotes the speed of light.

It seems that the condition (11) for complete coherence at a single frequency is more realistic than the "global" condition (5) for complete coherence of the total field. In particular it can be expected that the condition (12) will correctly describe the form of the cross-spectral density function for frequency components at which a laser is oscillating.

It would be of practical interest to carry our analysis one step further, going from a single frequency component to a very narrow band of frequencies around the mean frequency. It is likely that such an extention would provide the general form of the mutual coherence function of a highly coherent, quasi-monochromatic field. We plan to consider this problem as part of the research that we are currently carrying out under Grant No. DAAG29-77-G-0006. We are postponing the

publication of the results that we just discussed until it becomes clear whether further significant results in this direction can be obtained.

1. For a fuller discussion of the underlying concepts, see, for example, M. Born and E. Wolf, Principles of Optics, 5th Ed. (Pergamon Press, Oxford, 1975), Chapter X.

2. They are discussed, for example, in footnote 14 of ref. 3 given below and also in R. Barakat, "Theorem in Coherence Theory", J. Opt. Soc. Amer., 56, 739 (1966).

3. C. L. Mehta, E. Wolf and A. P. Balachandran, "Some Theorems on the Unimodular Complex Degree of Optical Coherence", J. Math. Phys. 7, 133 (1966).

4. L. Mandel and E. Wolf, "Spectral Coherence and the Concept of Cross-Spectral Purity", J. Opt. Soc. Amer., 66, 529 (1976).

(b) The Range of Validity of Tatarskii's Relationship between the Specific Intensity of Radiation and a Second-Order Correlation Function of Statistical Wavefields

In recent years several attempts have been made to elucidate the relationship between the specific intensity of radiation and various second-order correlation functions of the wavefield. In particular, Tatarskii[5] proposed a relationship of this kind, that appears to follow naturally from his calculations relating to the wave propagation in random media, based on quantum field-theoretical methods. In the course of his calculations, Tatarskii made a certain assumption relating to the spectral representation of the cross- spectral density function. However the physical significance of that assumption was not discussed by Tatarskii.

We have critically examined Tatarskii's analysis and were lead to the conclusion that Tatarskii's assumption restricts rather drastically the range of validity of his theory. More specifically we have shown that in free space, his theory is strictly valid only if the wavefield is statistically homogeneous in the statistical sense, i.e., if the cross-spectral density function $W(\underline{r}_1, \underline{r}_2, \nu)$ of the field depends on the two positional variables $\underline{r}_1$ and $\underline{r}_2$ through the difference $\underline{R} = \underline{r}_1 - \underline{r}_2$ only.

We have also examined an alternative definition of the specific intensity introduced more recently by Ovchinnikov and Tatarskii[6] and found that this definition is not consistent with a basic postulate of the theory of radiative energy transfer (the so-called flux relation). It would seem that no definition of the specific intensity of radiation can be given on the basis of the statistical wave theory that would satisfy, for every stationary ensemble of free fields, all the properties postulated by the phenomenological theory of radiative energy transfer. This opinion is supported by some considerations outlined elsewhere[7] by the Principal Investigator on this grant.

A full account of this investigation is contained in a paper that has been accepted for publication in the Journal of the Optical Society of America. The Abstract of the paper is given on page 10 of this report.

We have also presented the main results in a paper contributed at a meeting of the Optical Society of America. The Abstract is given on page 11.

5. V. I. Tatarskii, The Effects of the Turbulant Atmosphere on Wave Propagation, (U.S. Department of Commerce, Washington, D. C., 1971), Sec. 63.

6. G. I. Ovchinnikov and V. I. Tatarskii, "On the Problem of the Relationship between Coherence Theory and the Radiation-Transfer Equation", Radiophysics and Quantum Electronics, 15, 1087 (1972).

7. E. Wolf, "New Theory of Radiative Energy Transfer in Free Electromagnetic Fields", Phys. Rev., D, 13, 869 (1976), Sec. 6.

3. PUBLICATION AND ABSTRACT

The following article that describes research carried out under this grant has been accepted for publication by the Journal of the Optical Society of America:

E. Collett, J. T. Foley and E. Wolf: "On an Investigation of Tatarskii into the Relationship between Coherence Theory and the Theory of Radiative Energy Transfer".

ABSTRACT

In an investigation into the foundations of the theory of radiative energy transfer, V. I. Tatarskii (1971) postulated a certain relationship between the specific intensity of radiation and a second order coherence function of the wave-field. It is shown in the present paper that for propagation in free space, Tatarskii's assumption restricts the validity of his theory to fields that are statistically homogeneous.

A copy of the full manuscript is available on request from Prof. E. Wolf, Department of Physics and Astronomy, University of Rochester, Rochester, New York, 14627.

4. PAPER PRESENTED AT A SCIENTIFIC MEETING

Some of the results of this research were presented in a paper at the Annual Meeting of the Optical Society of America held in Tucson, Arizona in October, 1976:

E. Wolf, E. Collett and J. T. Foley: "On Tatarskii's Investigations into the Foundations of the Theory of Radiative Energy Transfer".

ABSTRACT

Tatarskii put forward an elegant theory,[1] concerning the relationship between the mutual coherence function and the specific intensity of radiation. The theory contains a hypothesis, relating to the spectral representation of the coherence function. In the present paper it will be shown that when the medium is homogeneous, Tatarskii's hypothesis restricts the validity of his theory to wave fields that are strictly homogeneous in the statistical sense. Some aspects and limitations of a modified theory, due to Ovchinnikov and Tatarskii[2] will also be discussed. Comparisons with another theory that was recently put forward[3] will also be made.

1. V. I. Tatarskii, The Effects of the Turbulant Atmosphere on Wave Propagation (U.S. Department of Commerce, Washington, D.C., 1971), Sec. 63.
2. G. I. Ovchinnikov and V. I. Tatarskii, Radiophys. Quantum Electron. 15, 1087 (1972).
3. E. Wolf, Phys. Rev. D 13, 869 (1976).

[Abstract MF 11, J. Opt. Soc. Amer., 66, 1065 (1976)].

5. PERSONNEL

The following persons associated with the University of Rochester, assisted in the research under this grant:

E. WOLF, Professor of Physics, Principal Investigator

J. T. FOLEY, Research Assistant

M. S. ZUBAIRY, Research Assistant

In addition Dr. Edward COLLETT of the U. S. Army Electronics Command, Fort Monmouth, New Jersy, collaborated on some of the research.